# Black hole singularities across phase transitions

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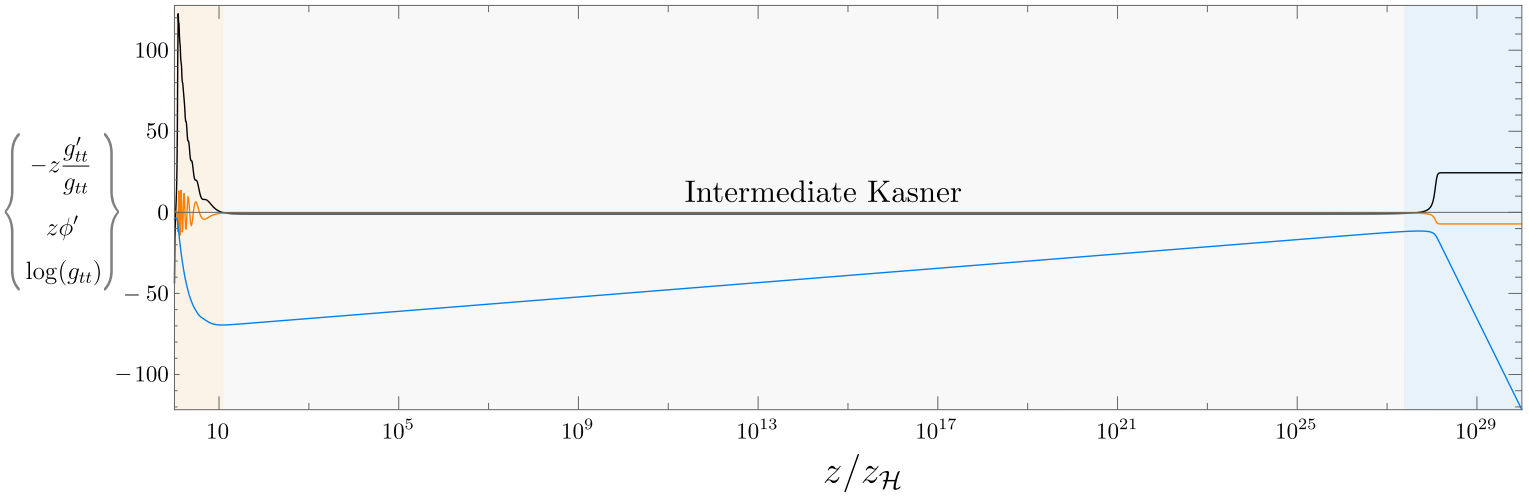
based on
2108.04554, work with Yan Liu(刘焱), Avinash Raju
2301.01468, work with Ling-Long Gao(高凌龙), Yan Liu

- Black hole interior and exterior are causally disconnected in classical gravity.
- Physics of black hole interior are important for understanding quantum gravity.
- Physical quantities of boundary field theory can be used to probe the interior of black hole in the AdS/CFT framework.
- Studying the interior of black hole is important even in classical gravity.




- No inner-horizon
- Collapse of Einstein-Rosen bridge
- Josephson oscillation
- Kasner singularity with Kasner inversion

[S.A. Hartnoll, G.T. Horowitz, J. Kruthoff, J.E. Santos, 2008.12786]

Kasner singularity:

$$ds^2 \sim -d\tau^2 + \tau^{2p_t} dt^2 + \tau^{2p_i} (dx^i)^2, \phi \sim p_\phi \log \tau$$

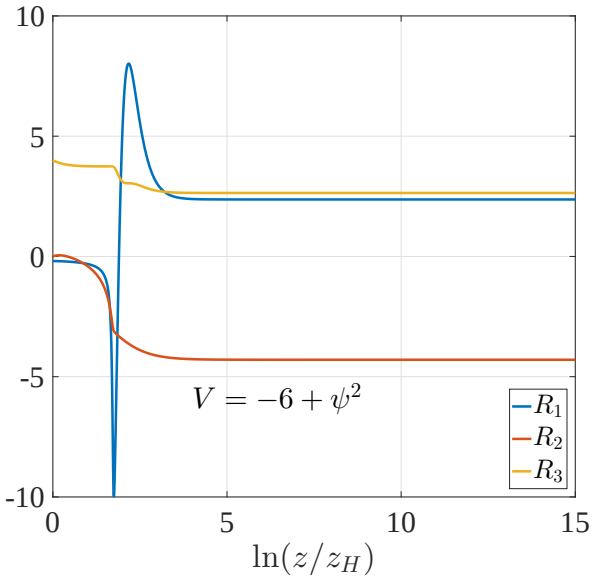
where Kasner exponents $p_t$ , $p_i$ , $p_\phi$ obey

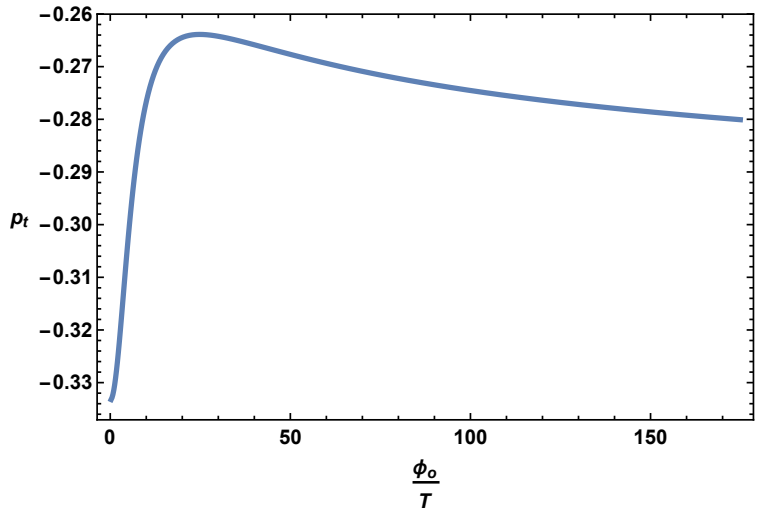
$$p_t + \sum_i p_i = 1\,, \;\; p_t^2 + \sum_i p_i^2 + p_\phi^2 = 1$$

Phase transition of holographic superconductor: R-N black hole(timelike singularity) $\rightarrow$ hairy black hole(spacelike Kasner singularity)

What happens to singularity during phase transition in CFT?

In Einstein-Scalar theory with polynomial scalar potential, Kasner singularities is stable

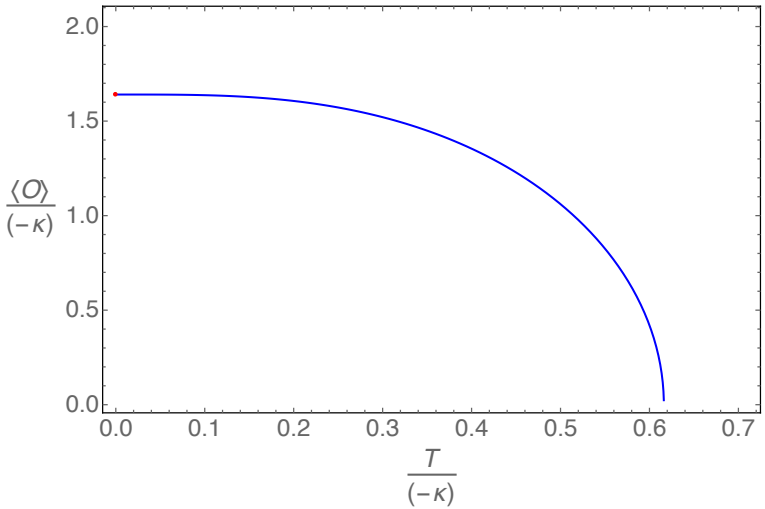


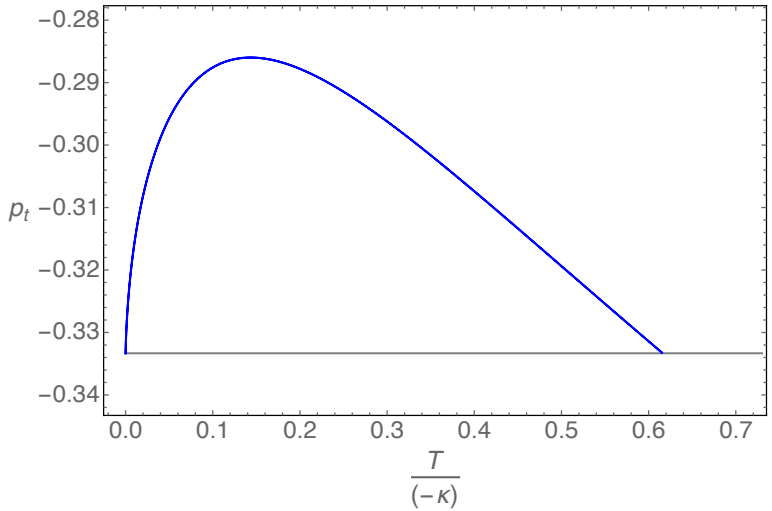




[R.G Cai, L. Li, R.Q. Yang, 2009.05520]
[A.Frenkel, S.A. Hartnoll, J. Kruthoff, Z.D. Shi, 2004.01192]

# Holographic insulator

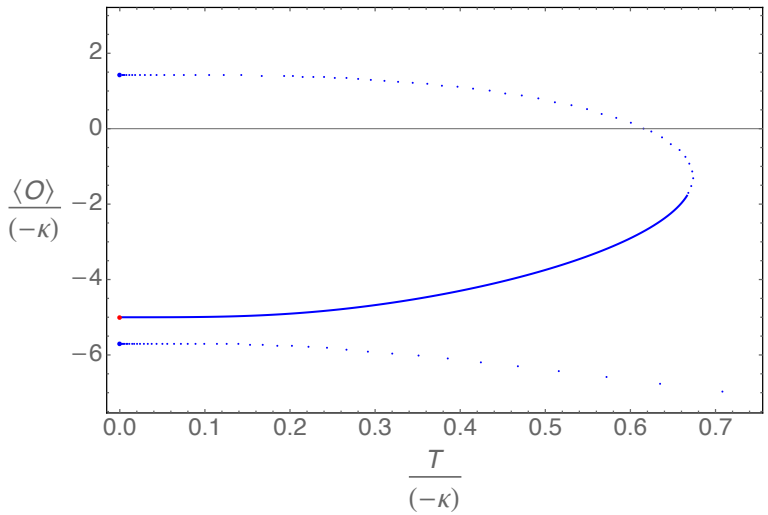
Second order phase transition: the Kasner exponents change continuously but not smoothly

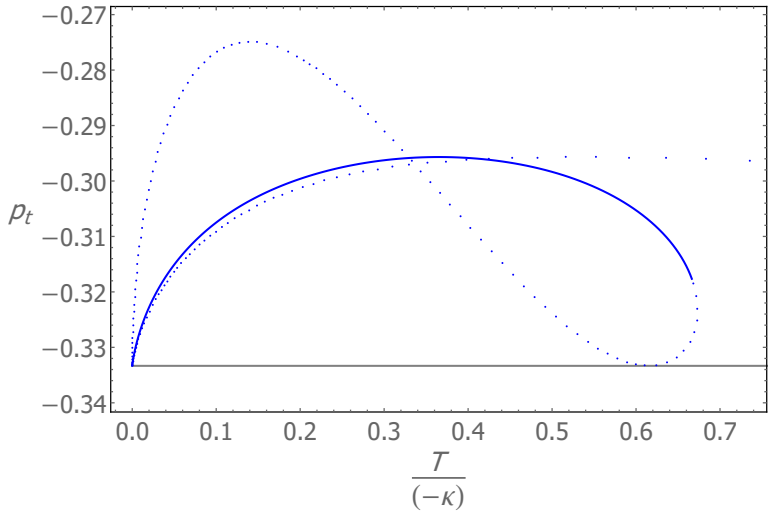


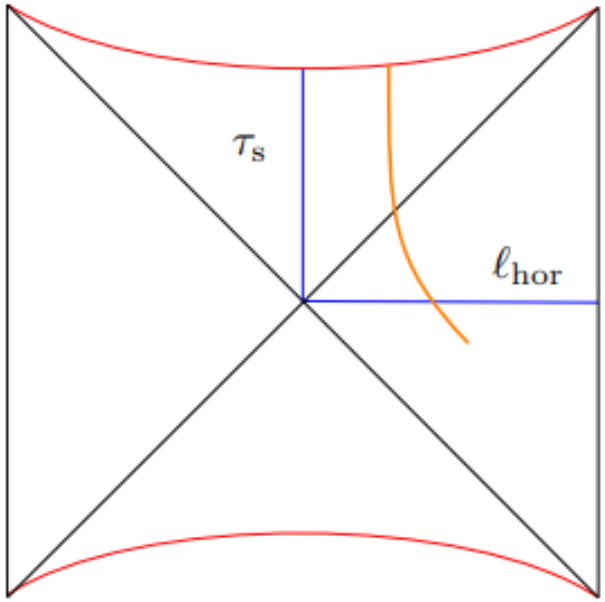

[E. Mefford, G.T. Horowitz, 1406.4188]
[Y. Liu, HDL, A. Raju, 2108.04554]

For an asymmetric potential, there is a first order phase transition and the Kasner exponents have a jump.



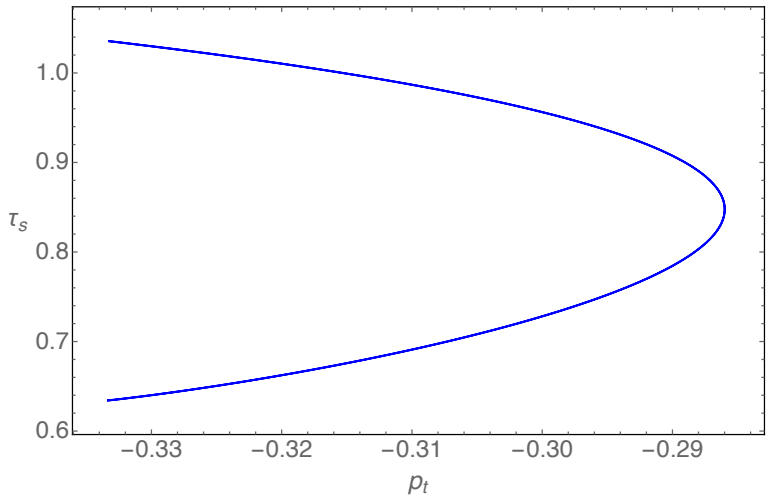
Proper time can be encoded in the CFT

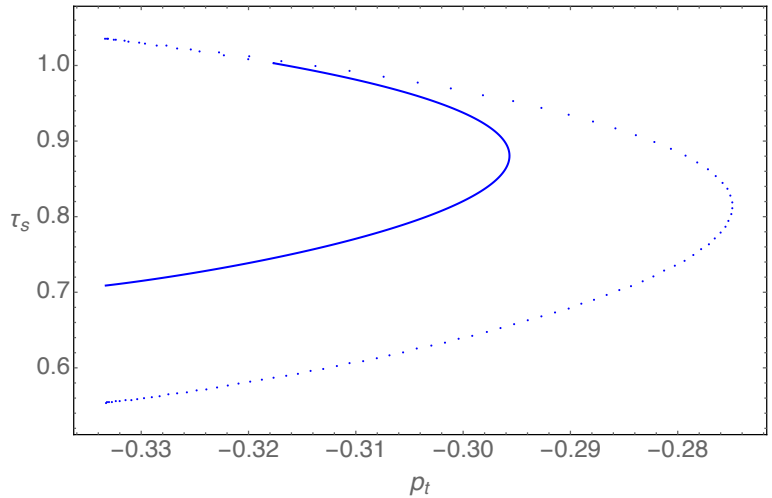



$$\langle O \rangle \sim \exp[-im\tau_s - ml_{hor}] \tag{1}$$

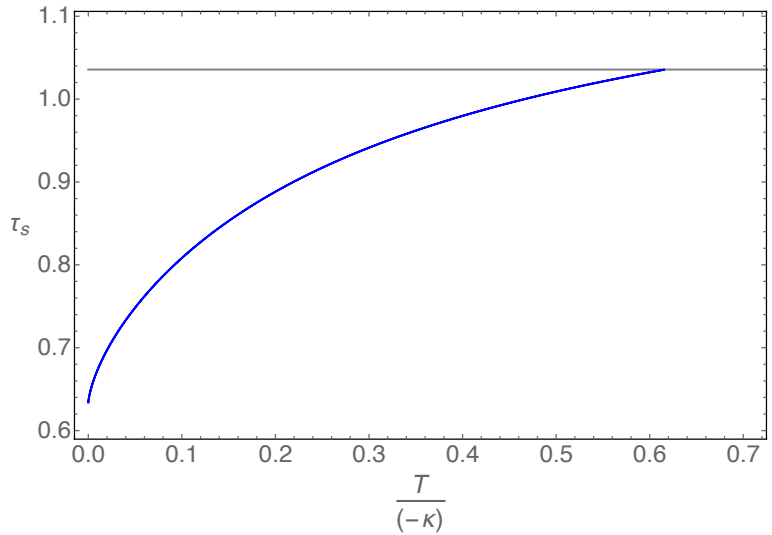
[M. Grinberg, J. Maldacena, 2011.01004]

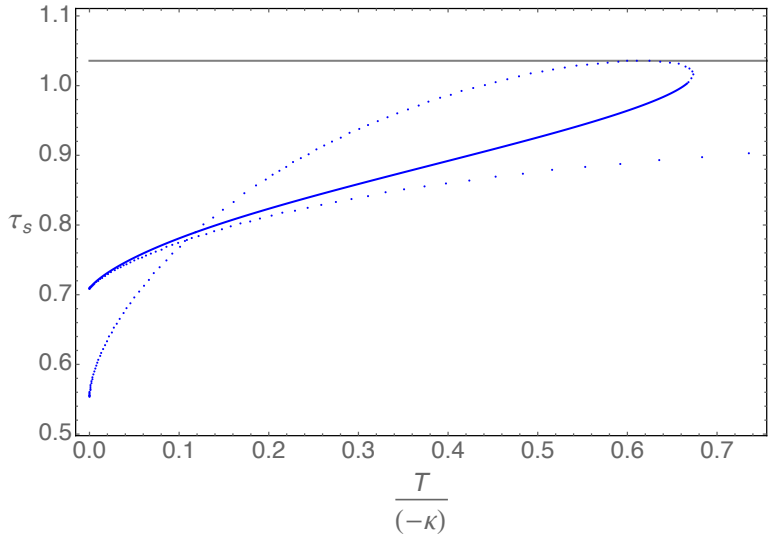
# Probe Kasner exponents of singularity



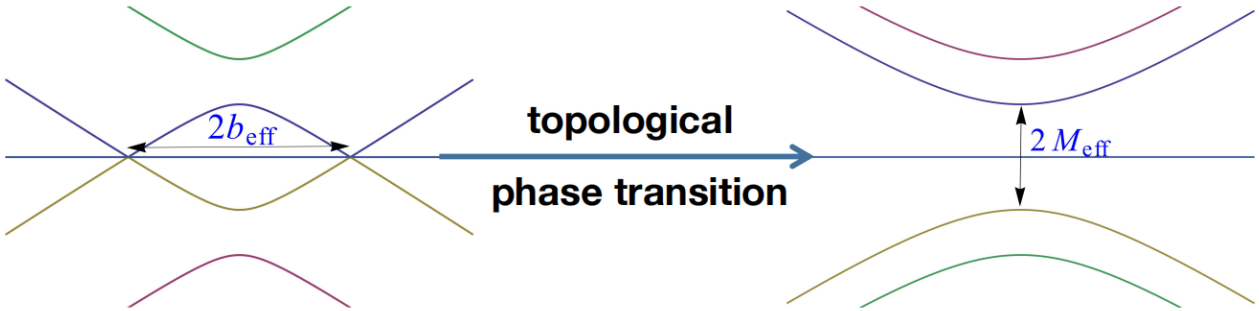
# Probe phase transition



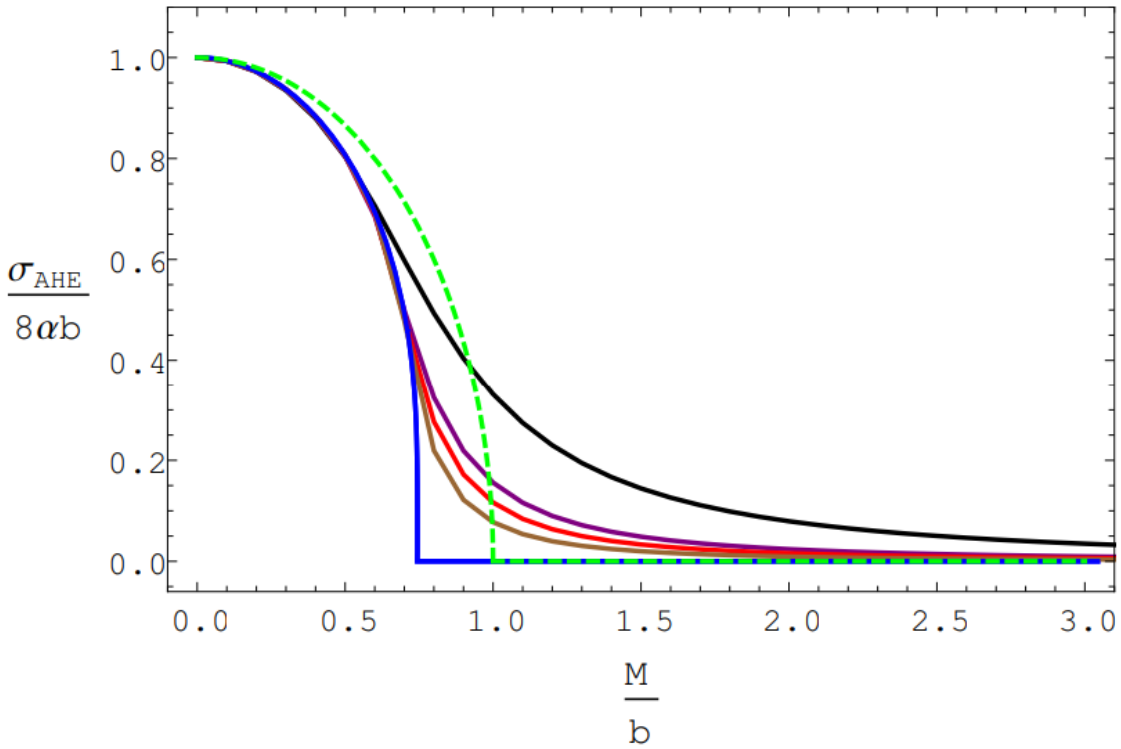
# Topological semimetal

There is a topological phase transition witch is beyond Landau paradigm.



# Holographic Weyl semimetal

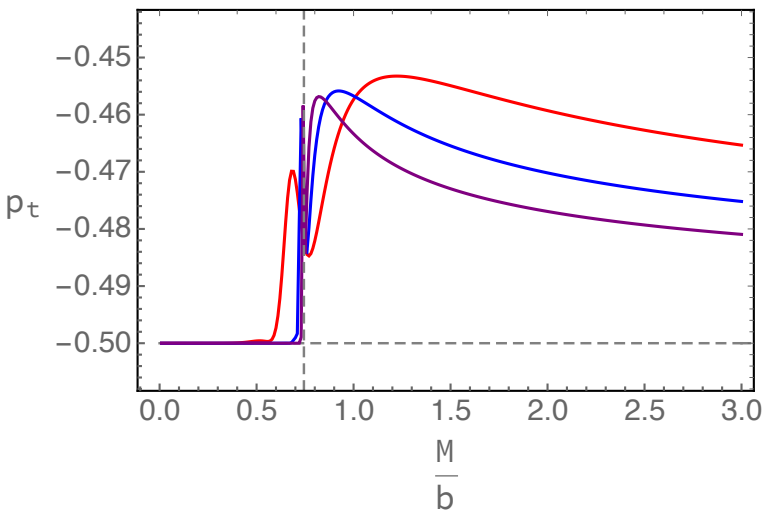
Anomalous Hall conductivity serves as an order parameter

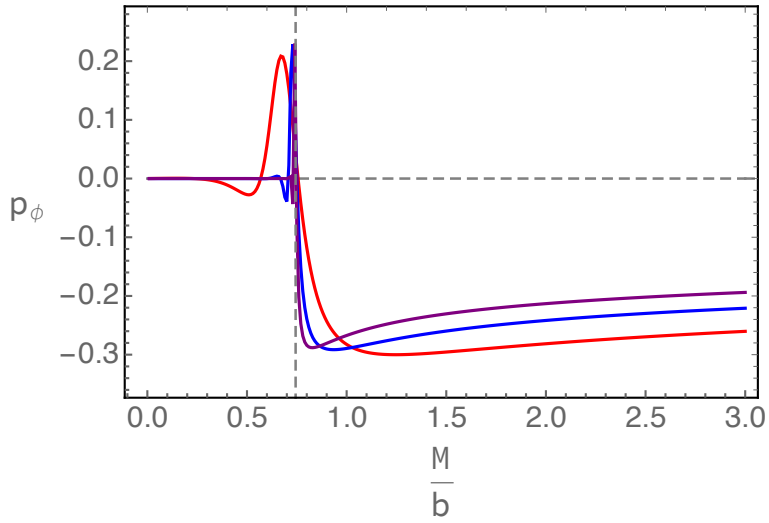



[K. Landsteiner, Y. Liu, Y.W. Sun, 1511.05505]

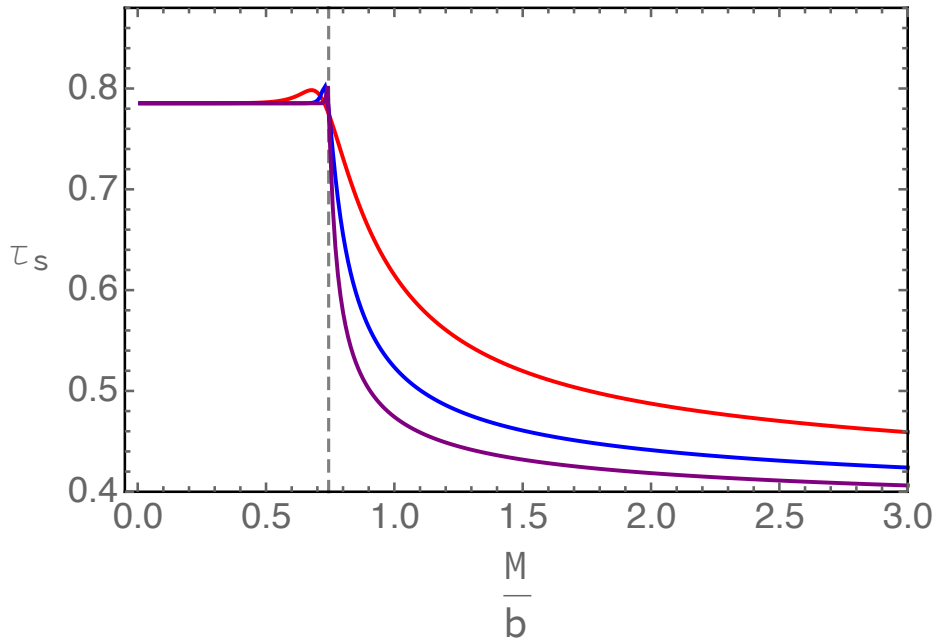
In topological phase,all Kasner exponents are

- contant
- the same values as Schwarzschild black hole



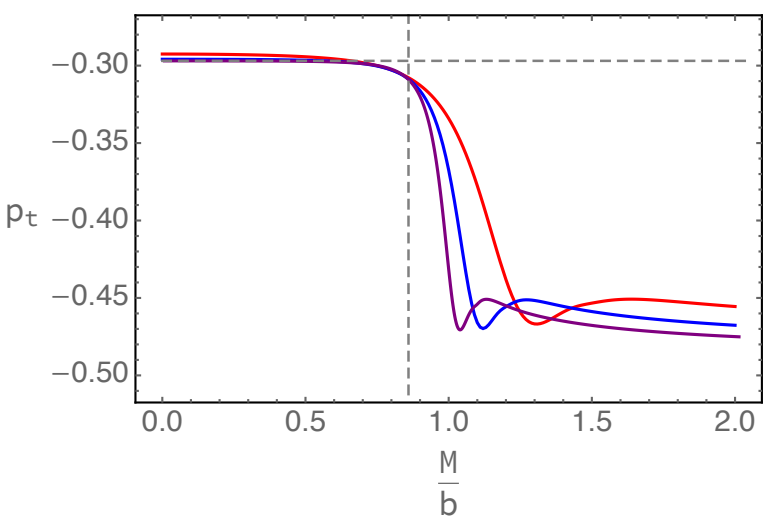
Proper time is also a constant and has the same value as Schwarzschild black hole in topological phase.

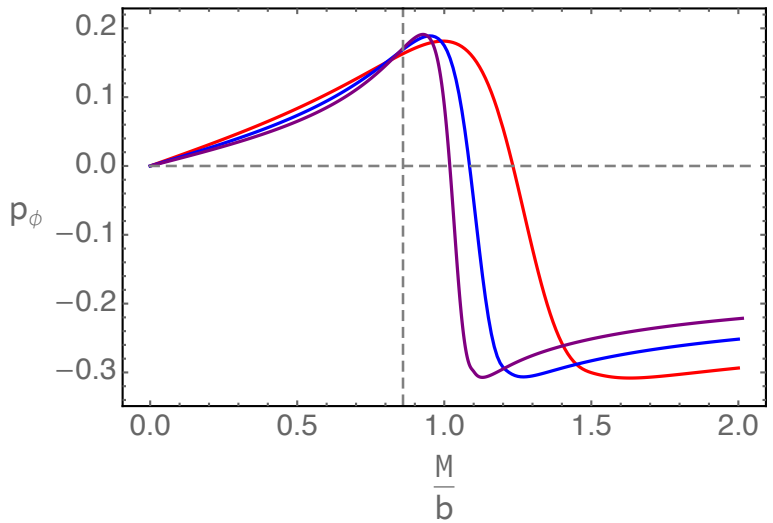


# Holographic nodal line semimetal

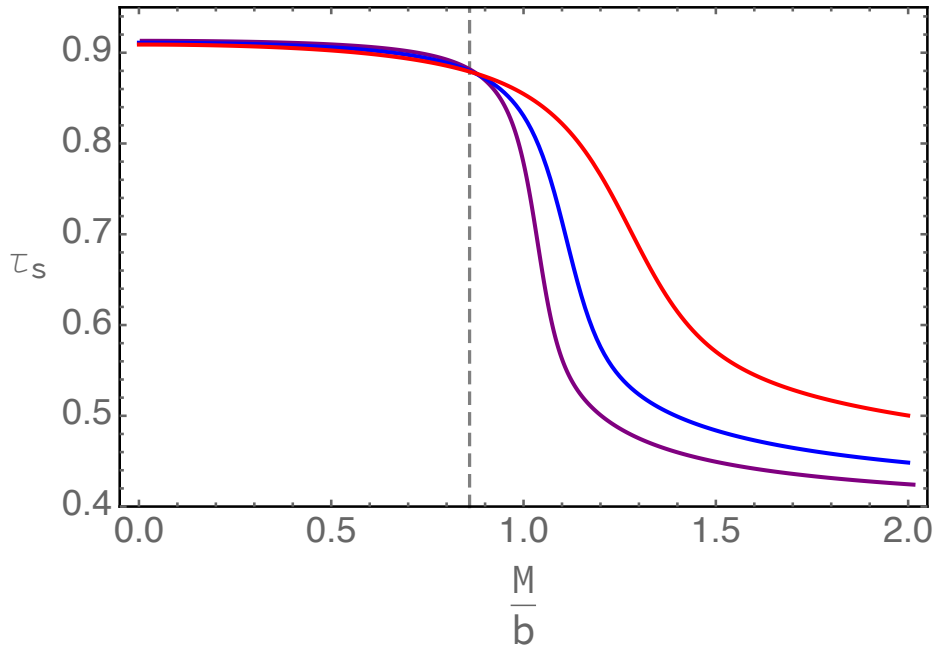
In topological phase

- Kasner exponents of geometry are constant
- matter fields strongly backreact to the IR geometry



Proper time is a constant in topological phase.



# Conclusion

- Second order phase transition: the change of Kasner exponents are continuous but not smooth
- First order phase transition: the change of Kasner exponents are discontinuous
- Topological phase transition: Kasner exponents have some topological behaviors
- Thermal one point function can be used to probe phase transition and serve as the "order" parameter for semimetal
- Thermal one point function can be used to probe the Kasner exponents of Singularity.

Thank you!